The Effects of Sine-Squared Pulse Modulation Correlated Noises on Stochastic Resonance in Single-Mode Laser

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By means of the linear approximation method, the output intensity power and signal-to-noise ratio (SNR) of a single-mode laser driven by sine-squared pulse modulation correlated noise are calculated. The effects of amplitude B, period T, and width τ on the resonance curves of SNR to the pump noise intensities and quantum noise intensities of pulse are discussed, and it is found that the SNR shows a stochastic resonance with the varying of pulse width τ.

Key words: Single-Mode Laser; Stochastic Resonance; Noises.
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1. Introduction

Since 1988, when McNamara et al. firstly observed that the output signal-to-noise ratio (SNR) exhibited a maximum versus the input noise intensity in a ring laser, i.e. shows stochastic resonance (SR) [1], it has attracted wide interests of investigation in optical systems. Many powerful approaches were presented to solve the laser models, some of them were effectively used to seek SR in laser systems. In recent decades, SR in the single-mode laser has been studied largely by means of the linear approximation method. For example, Cao et al. have studied the dynamic property and statistical fluctuation of a single-mode laser system driven by both pump noise and quantum noise with cross-correlation between the real and imaginary parts [2, 3]. The same authors have studied the influence of the input different signal and the noise on the characteristic of SR in a single-mode laser system [4 – 9]. In laser communication, when the laser is modulated by a signal, the noise in the laser is modulated too, and the modulated noise takes great effect on the statistical properties and the SR in the laser. Chen and Wang studied the effects of period modulation of the noise correlation intensity and bias signal modulation of the noise on SR in a single-mode laser [10, 11]. Zhang et al. found the SR in single-mode laser with signal-modulated noise or frequency modulated signals, too [12 – 14]. In their studies, the modulated signals are sine (or cosine) periodical signals. However, the laser is mainly used as carrier of pulse signals in modern laser communications, so the noise may be modulated by a pulse signal, but there are few reports discussing the SR of a laser with pulse modulation noise. In this paper, we study the SR in a single-mode laser driven by the sine-squared pulse modulation correlated noise, and discuss the effects of the amplitude, the period, and width of the pulse on the SNR.

2. The Output Intensity Power and SNR of Single-Mode Laser Model

The corresponding Langevin equation of the intensity for the gain-noise model of a single-mode laser is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2\Gamma}{1+\beta I}I + D + \frac{2I}{1+\beta I}\xi(t) + 2\sqrt{I}\eta(t) + A\cos(\Omega t), \tag{1}$$

where I is the laser intensity, $\beta = \tilde{A}/\Gamma$, $\tilde{A}$ and Γ represent the self-saturation and gain coefficients, respectively, K is the loss coefficient; A and Ω express the amplitude and frequency of the periodical signal; $\xi(t)$ and $\eta(t)$ are pump noise and quantum noise, the statistical properties of them are characterized

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by

$$\begin{aligned}
&\langle \xi(t)\rangle = \langle \eta(t)\rangle = 0,\\
&\langle \xi(t)\xi(t')\rangle = Q\delta(t-t'),\\
&\langle \eta(t)\eta(t')\rangle = D\delta(t-t'),\\
&\langle \xi(t)\eta(t')\rangle = \langle \xi(t')\eta(t)\rangle\\
&\qquad = \lambda f(t)\sqrt{QD}\delta(t-t'),\ (-1<\lambda<1).
\end{aligned} \tag{2}$$

In above equations, Q and D are the intensities of the pump noise and quantum noise, respectively, and λ is the noise correlation coefficient. $f(t)$ represents the sine-squared pulse modulation signal, which is expressed by

$$f(t) = \begin{cases} B\sin^2\left(\dfrac{\pi(t-nT)}{2\tau}\right) & t \in (nT, nT+2\tau) \\ & (n=0,1,2,3\ldots), \\ 0 & t\in(nT+2\tau,(n+1)T) \\ & (n=0,1,2,3\ldots), \end{cases} \tag{3}$$

where B, T, and τ denote the amplitude, the period, and the width of the pulse signal, respectively.

We linearized (1) around the deterministic steady-state intensity $I_0 = \frac{(\Gamma-K)}{\beta K}$ and let

$$I = I_0 + \varepsilon(t), \tag{4}$$

where $\varepsilon(t)$ is the perturbation term substituted into (1), thus the linearized equation of the laser intensity is obtained as

$$\begin{aligned}
\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t} &= -\gamma\varepsilon(t) + D + \frac{2I_0}{1+\beta I_0}\xi(t)\\
&\quad + 2\sqrt{I_0}\eta(t) + A\cos(\Omega t),
\end{aligned} \tag{5}$$

where $\gamma = \frac{2K(\Gamma-K)}{\Gamma}$ is the damping coefficient.

By virtue of straight forward integration of (5) and according to the definition of the mean laser intensity correlation function

$$\begin{aligned}
C(t') &= \lim_{t\to\infty}\overline{\langle I(t+t')I(t)\rangle}\\
&= \lim_{t\to\infty}\frac{\Omega}{2\pi}\int_t^{t+\frac{2\pi}{\Omega}}\langle I(t+t')I(t)\rangle\,\mathrm{d}t,
\end{aligned} \tag{6}$$

we obtain

$$\begin{aligned}
C(t') &= \left[\frac{2I_0^{(3/2)}\lambda\sqrt{DQ}}{\gamma(1+\beta I_0)}\frac{(1-\exp(-4\gamma\tau))B\exp(2\gamma T)}{(\exp(2\gamma T)-1)(4\gamma^2\tau^2+\pi^2)}\right.\\
&\quad \left. + \frac{2I_0^2 Q}{\gamma(1+\beta I_0)^2} + \frac{2I_0 D}{\gamma}\right]\exp(-\gamma t')\\
&\quad + \frac{A^2}{2(\gamma^2+\Omega^2)}\cos(\Omega t') + I_0^2 + \frac{D^2}{\gamma^2} + \frac{2I_0 D}{\gamma}.
\end{aligned} \tag{7}$$

By the Fourier transform of (7), we finally obtain the expression of the output power spectrum

$$S(\omega) = \int_{-\infty}^{\infty} C(t')\exp(\mathrm{i}\omega t')\,\mathrm{d}t' = S_1(\omega) + S_2(\omega), \tag{8}$$

where $S_1(\omega)$is the output spectrum of the signal,

$$S_1(\omega) = \frac{A^2}{2(\gamma^2+\Omega^2)}\pi[\delta(\omega-\Omega)+\delta(\omega+\Omega)]. \tag{9}$$

The total output signal power is

$$P_\mathrm{s} = \int_0^\infty S_1(\omega)\,\mathrm{d}\omega = \frac{\pi A^2}{2(\gamma^2+\Omega^2)}. \tag{10}$$

$S_2(\omega)$ is the output noise power spectrum,

$$\begin{aligned}
S_2(\omega) &= \left[\frac{2I_0^{(3/2)}\lambda\sqrt{DQ}}{\gamma(1+\beta I_0)}\frac{(1-\exp(-4\gamma\tau))B\exp(2\gamma T)}{(\exp(2\gamma T)-1)(4\gamma^2\tau^2+\pi^2)}\right.\\
&\quad \left. + \frac{2I_0^2 Q}{\gamma(1+\beta I_0)^2} + \frac{2I_0 D}{\gamma}\right]\frac{2\gamma}{\gamma^2+\omega^2}.
\end{aligned} \tag{11}$$

The output total noise power reads

$$S_2 = S_2(\omega=\Omega). \tag{12}$$

The signal-to-noise ratio (SNR) is defined as the ratio of the output power of the signal P_s and the noise power spectrum S_2:

$$\mathrm{SNR} = \frac{P_\mathrm{s}}{S_2}. \tag{13}$$

The SNR may be obtained by

$$\begin{aligned}
\mathrm{SNR} &= [\pi A^2]\\
&\quad \cdot\left[\frac{4I_0^{(3/2)}\lambda\sqrt{DQ}}{(1+\beta I_0)}\frac{(1-\exp(-4\gamma\tau))B\exp(2\gamma T)}{(\exp(2\gamma T)-1)(4\gamma^2\tau^2+\pi^2)}\right.\\
&\quad \left. + \frac{4I_0^2 Q}{(1+\beta I_0)^2} + 4I_0 D\right]^{-1}.
\end{aligned} \tag{14}$$

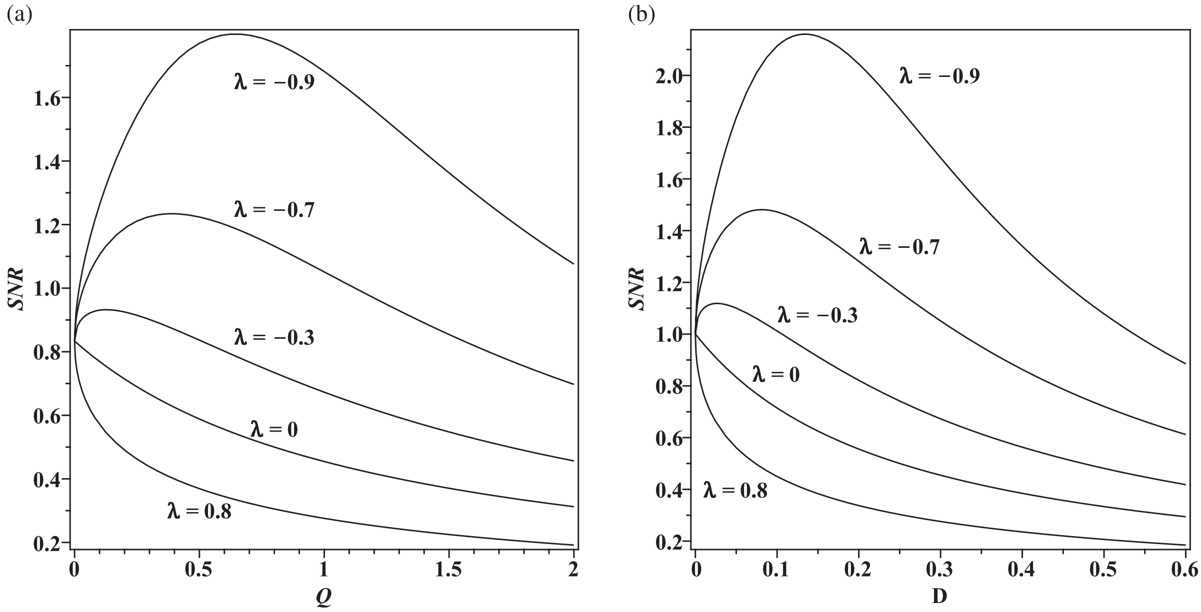

Fig. 1. SNR as a function of the noise intensity Q and D for different values of the cross-correlation coefficient λ. Here the parameters adopted are: $\beta = 1$, $I_0 = 1$, $\gamma = 1$, $T = 2$, $B = 2$, $\tau = 0.5$, and (a) $D = 0.3$, (b) $Q = 1$.

3. The Stochastic Resonance of Output SNR

According to (14), in our model, the phenomena of SR exist in the curves of the SNR versus the pump noise intensities Q, the quantum noise intensities D, and the pulse width τ. The effect of sine-squared pulse modulation noise on SR is described as follows.

3.1. Effect of Noise Correlation Coefficient λ on SNR-Q *and* SNR-D *Curves*

Using (14), the curve of the SNR as a function of the noise intensities Q and D with the cross-correlation coefficient λ are plotted in Figure 1. It is obtain that the SNR decreases monotonically with Q and D in the case of $\lambda \geq 0$, whereas the SNR-Q and SNR-D curves exhibit one resonance peak in the case of $\lambda < 0$. Its maximum increases with the increase of $|\lambda|$, and the position moves towards the increased Q and D. This is the conventional form of SR, and it is the same result as without pulse modulation.

3.2. Effect of Pulse Amplitude B on SNR-Q *and* SNR-D *Curves*

Figure 2 depicts the dependence of the SNR on Q and D. We can see that the SNR is monotone increasing with Q or D when the value of the pulse amplitude B is smaller than zero. The resonance peak of SNR-Q and SNR-D curves appears in the case of $B > 0$, and the position of the peak moves towards the increased noise intensity and its intensity grows with the increases of B. Hence, there is the same effect observed on the SNR as for the cross-correlation coefficient λ modulating the pulse amplitude B. This demonstrates that the amplitude B of the pulse mainly affects the noise correlation intensity, and the larger the pulse amplitude B, the stronger is the noise correlation intensity.

3.3. Effect of Ratio of Pulse Width to Period $(2\tau/T)$ on SNR-Q *and* SNR-D *Curves*

In order to discuss the effect of pulse duration on the SNR, we choose the ratio of pulse width to period $(2\tau/T)$ as parameter. It expresses the relative time of pulse driving in one period. The curves of SNR-Q and SNR-D with the change of $2\tau/T$ are shown in Figure 3. It can be seen that the height of the resonance peak gradually grows and its position moves towards the increased Q and D when $2\tau/T$ increases. However, the $2\tau/T$ greatly affects the SNR only when it takes a large value $(2\tau/T > 0.5)$. This illustrates that, under the condition of τ taking a fixed value, the larger the value of $2\tau/T$, the longer is the time of pulse

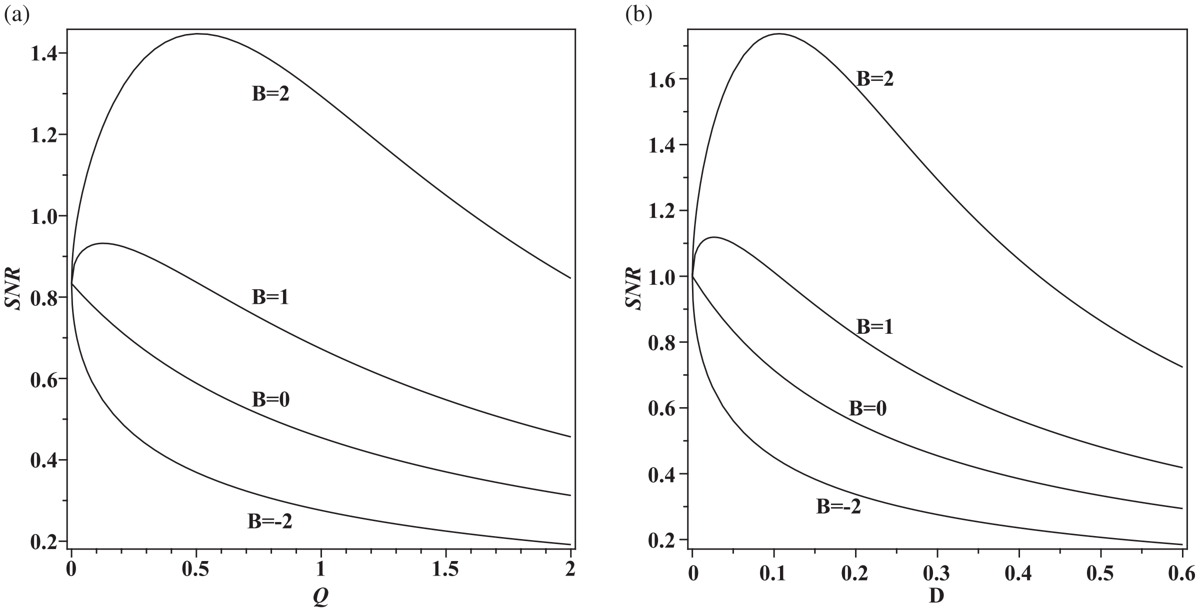

Fig. 2. SNR as a function of the noise intensity Q and D for different values of pulse amplitude B. Here the parameters adopted are: $\beta = 1$, $I_0 = 1$, $\gamma = 1$, $T = 2$, $\lambda = -0.8$, $\tau = 0.5$, and (a) $D = 0.3$, (b) $Q = 1$.

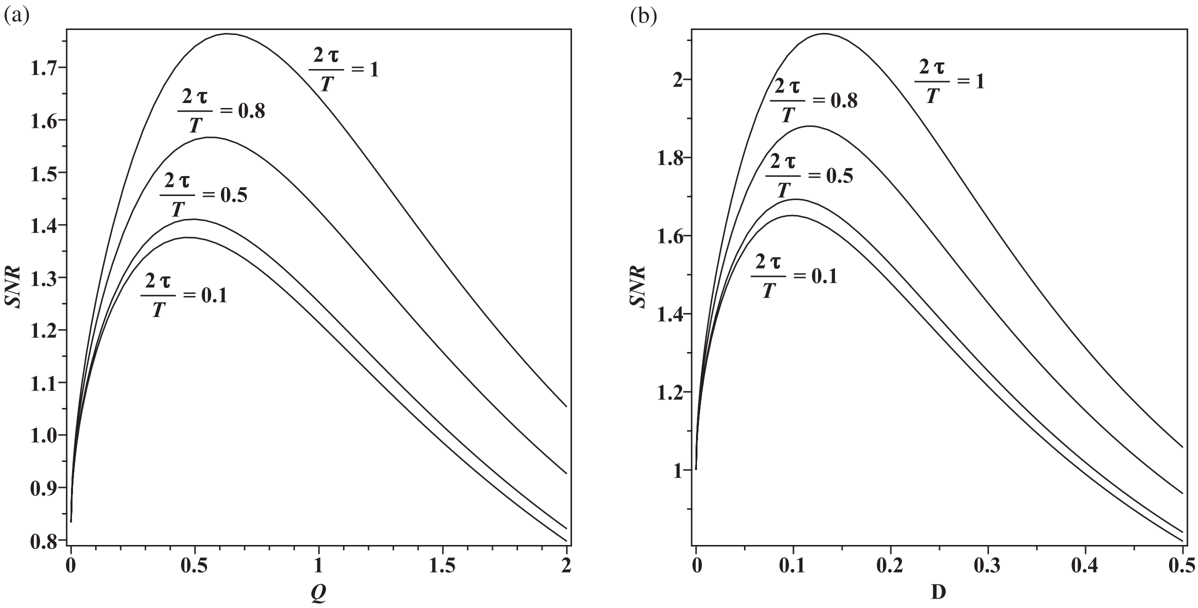

Fig. 3. SNR as a function of the noise intensity Q and D for different values of ratios of pulse width to period $(2\tau/T)$. Here the parameters adopted are: $\beta = 1$, $I_0 = 1$, $\gamma = 1$, $B = 2$, $\lambda = -0.8$, $\tau = 0.5$, and (a) $D = 0.3$, (b) $Q = 1$.

driving, and pulse modulation makes the noise correlation intensity stronger. So there is a great effect of pulse modulation on the SNR when $2\tau/T$ takes a large value.

3.4. Effect of Pulse Width on SNR

For the effect of the pulse width τ on the SNR, we choose τ as parameter. Figure 4a depicts the depen-

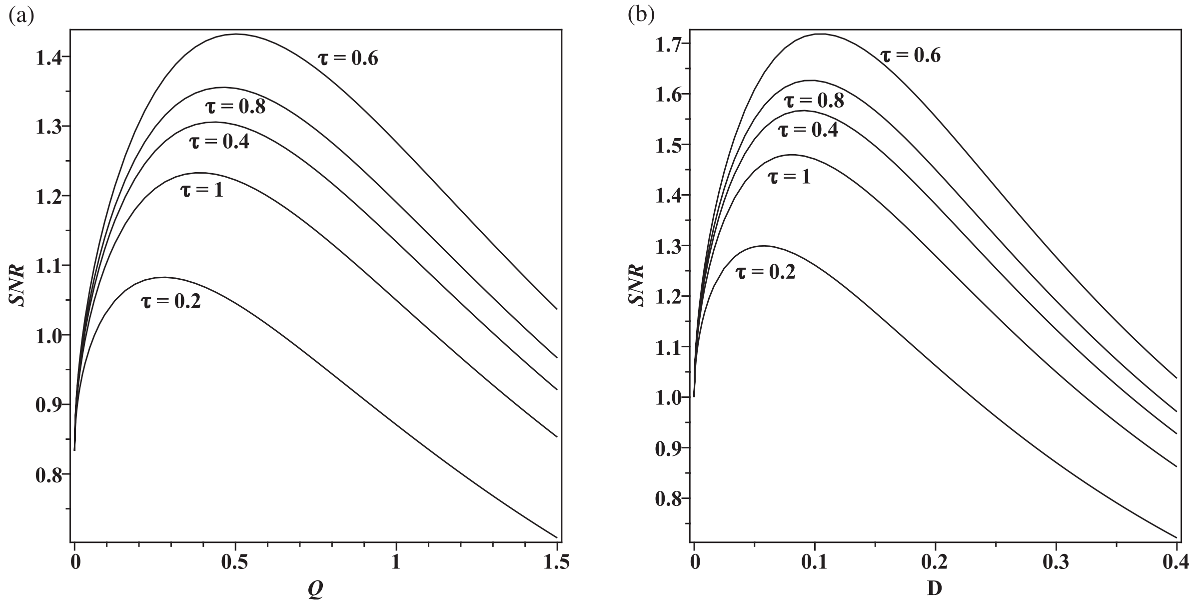

Fig. 4. SNR as a function of the noise intensity Q and D for different values of pulse width τ. Here the parameters adopted are: $\beta = 1$, $I_0 = 1$, $\gamma = 1$, $B = 2$, $\lambda = -0.8$, $T = 2$, and (a) $D = 0.3$, (b) $Q = 1$.

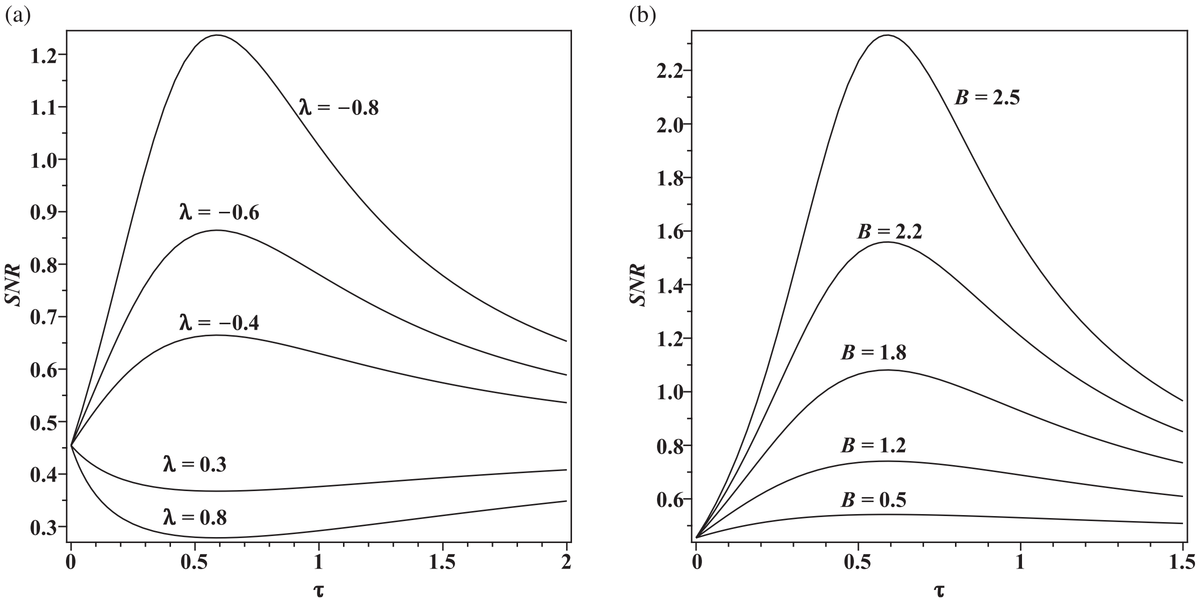

Fig. 5. SNR as a function of the pulse width τ for different values of cross-correlation coefficient λ and pulse amplitude B. Here the parameters adopted are: $\beta = 1$, $I_0 = 1$, $\gamma = 1$, $D = 0.3$, $Q = 1$, $T = 2$, and (a) $B = 2$, (b) $\lambda = -0.8$.

dence of the SNR on Q. We can see that the SNR-Q curve exhibits SR. As the pulse width τ increases, the intensity of the resonance peak grows and its position moves towards the increased Q in the case of $\tau < 0.6$. However within the range $0.6 < \tau < 1$, as τ increases, the maximum value of the resonance peak shrinks and its position moves towards the decreased Q. Figure 4b depicts the dependence of the SNR on D. It shows that the same tendency appears as in the corresponding part of Figure 4a.

The SNR as a function of pulse width τ for different values of the cross-correlation coefficient λ and pulse amplitude B are plotted in Figure 5a and Figure 5b, respectively. From Figure 5a, we can clearly see that in the SNR-τ curve SR appears in the case of $\lambda < 0$. As λ decreases, the height of the peak grows, but its position does not change. In the case of $\lambda > 0$, the SNR-τ curve exhibits one minimum, i.e. the SNR change shows suppression. As λ increases, the position of the curve moves downwards, and makes the suppression stronger. Figure 5b depicts the dependence of SNR on τ. SR appears when the pulse amplitude B takes a larger value, and the larger the pulse amplitude B, the higher is the resonance peak, but the position of the peak does not depend on B. When B takes small values, the peak disappears as shown in the figure, and there is hardly an effect of the pulse width τ on the SNR.

4. Summary and Discussion

In summary, in the single-mode laser model driven by a sine-squared pulse modulation correlated noise and periodic signal, only when the pulse amplitude B takes larger value and the cross-correlation coefficient R is smaller than zero, SR occurs in curves of SNR-Q, SNR-D, and SNR-τ, and with an increase of the pulse amplitude B, the height of the resonance peak becomes higher. The pulse width τ and the pulse period T affect the value and the position of resonance peak of SNR-Q and SNR-D curves, but don't affect the occurrence of SR.

Acknowledgements

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